**American University of Beirut**
***Department of Computer Science***
**CMPS 211 – Discrete Mathematics – Spring 13/14**

Please solve the following exercises and submit **BEFORE 9:00 am of Monday 31st, March.**

## Exercise 1 (10 points)

a) Find a formula for

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}$$

by examining the values of this expression for small values of n.

b) Prove the formula you conjectured in part (a).

## Exercise 2 (10 points)

Prove that 6 divides $n^3 - n$ whenever n is a nonnegative integer.

## Exercise 3 (10 points)

What is wrong with this "proof"?

"Theorem" For every positive integer n, $\sum_{i=1}^{n} i = \frac{\left(n+\frac{1}{2}\right)^2}{2}$.

Then $\sum_{i=1}^{k+1} i = \left(\sum_{i=1}^{k} i\right) + (k+1)$. By this inductive hypothesis, $\sum_{i=1}^{k+1} i = \frac{\left(k+\frac{1}{2}\right)^2}{2} + k + 1 = \frac{\left(k^2+k+\frac{1}{4}\right)}{2} + k + 1 = \frac{\left(k^2+3k+\frac{9}{4}\right)}{2} = \frac{\left(k+\frac{3}{2}\right)^2}{2} = \frac{\left[(k+1)+\frac{1}{2}\right]^2}{2}$, completing the inductive step.

## Exercise 4 (10 points)

Suppose that m and n are positive integers with m > n and f is a function from {1,2,...,m} to {1,2,...,n}. Use mathematical induction on the variable n to show that f is not one-to-one.

## Exercise 5 (20 points)

a) Determine which amounts of postage can be formed using just 4-cent and 7-cent stamps.

b) Prove your answer to (a) using the principle of mathematical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.

c) Prove your answer to (a) using strong induction. How does the inductive hypothesis in this proof differ from that in the inductive hypothesis for a proof using mathematical induction?

## Exercise 6 (10 points)

Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$, and so on. [*Hint:* For the inductive step, separately consider the case where k + 1 is even and where it is odd. When it is even, note that (k + 1)/2 is an integer.]

## Exercise 7 (10 points)

Prove using induction that $2n^2 - 10n + 4$ is non negative whenever $n$ is an integer and $n \geq 5$.

## Exercise 8 (10 points)

Prove using induction that if $A_1, A_2 \ldots A_n$ and $B$ are sets then

$$(A_1 \cap A_2 \cap \cdots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B).$$

## Exercise 9 (10 points)

Use mathematical induction to prove that 9 divides $n^3 + (n + 1)^3 + (n + 2)^3$ whenever n is a nonnegative integer.